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Viscoelastic response modelling of a pavement under moving load

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Abstract

This paper demonstrates the application of a generalized layered linear viscoelastic (LVE) analysis for estimating flexible pavements' structural response. The procedure is based on the Multi-Layered Elastic Theory (MLET) and the elastic-viscoelastic correspondence principle using a numerical inverse Laplace transform. A comparison of the direct layered viscoelastic responses with approximate solutions based on the elastic collocation method was also carried out. Furthermore, it is proposed to use the collocation method using LVE solutions at selected time durations in order to improve the accuracy of the elastic collocation method. The LVE collocation method was further extended for analysis of moving loads. The method was illustrated by analysing a pavement structure subjected to moving wheel loads of 30, 50, 60 and 80 kN using a Heavy Vehicle Simulator (HVS). The various responses (stresses and strains) in the pavement, at different pavement temperatures, were measured using various types of sensors installed in the structure. The LVE calculations agreed very well with the measurements.

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Keywords: Viscoelasticity; elastic-viscoelastic principle; layered elastic theory; accelerated pavement testing

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1. Introduction

In Mechanistic-Empirical (M-E) pavement design and performance prediction procedures a mathematical model is used to calculate the pavement responses under traffic loading and prevailing environmental conditions. The responses are then related to the performance of the pavement through transfer functions. Frequently the Multi-Layered Elastic Theory (MLET) (Burmister 1943, 1945) is employed to calculate the responses of layered systems under concentrated or distributed loading (Huang 1968, Bufler 1971, Maina and Matsui 2005, Khazanovich and Wang 2007, Erlingsson and Ahmed 2013).

It is well known that asphalt mixtures exhibit unique characteristics of both viscous and elastic properties, and hence are categorized as viscoelastic materials. Moreover, understanding the viscoelastic properties of asphalt mixtures are important to achieve performance-based structural design of bituminous layers (NCHRP 2004). Therefore it is important to extend the theory of MLET to account for the effect of viscoelasticity. This can be accomplished through the elastic-viscoelastic correspondence principle in which the elastic solution is used to derive solutions of linear viscoelastic (LVE) problems.

Erlingsson and Ahmed (2013) introduced a method to improve the computational performance of MLET. The MLET was developed for use in the M-E performance prediction program for flexible pavement structures. The main objective of the study presented here is to extend this work for problems involving LVE materials such as asphalt mixtures. Finally, as viscoelastic materials such as asphalt mixes possess time-dependent or rate sensitive stress-strain relations, their stress-strain relationship will change as the loading speed (or strain rate) changes. The viscoelastic solution is therefore extended to simulate the response of layered systems subjected to a moving wheel load.

2. Linear viscoelastic analyses

Flexible pavement structures consist of layers of finite thickness resting on half space. The top layers are bitumen bounded aggregate layers that are highly viscoelastic in nature when subjected to heavy traffic loading. Below are aggregate and soil layers that can be treated as elastic or elastic-plastic material layers. The mechanical response of such systems can be analysed with the aid of the MLET.

The axisymmetric layered elastic responses (stresses and displacements) under concentrated load can be obtained from a stress function ϕ that satisfies boundary and continuity conditions (Timoshenko and Goodier 1951, Huang 2004):

$$\begin{bmatrix} \sigma_z^c \\ \sigma_r^c \\ \sigma_t^c \\ \tau_{rz}^c \\ w^c \\ u^c \end{bmatrix} = \begin{bmatrix} (2-\nu)\dfrac{\partial}{\partial z} & -\dfrac{\partial^3}{\partial z^3} \\ \nu\dfrac{\partial}{\partial z} & -\dfrac{\partial}{\partial z}\dfrac{\partial^2}{\partial r^2} \\ \nu\dfrac{\partial}{\partial z} & -\dfrac{\partial}{\partial z}\dfrac{1}{r}\dfrac{\partial}{\partial r} \\ (1-\nu)\dfrac{\partial}{\partial r} & -\dfrac{\partial}{\partial r}\dfrac{\partial^2}{\partial z^2} \\ \dfrac{1+\nu}{E}(1-2\nu) & \dfrac{1+\nu}{E}\left(\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}\right) \\ 0 & -\dfrac{1+\nu}{E}\left(\dfrac{\partial^2}{\partial r \partial z}\right) \end{bmatrix} \begin{bmatrix} \nabla^2\phi \\ \phi \end{bmatrix} \tag{1}$$

where σ_z^c, σ_r^c and σ_t^c are normal stresses in vertical, radial and tangential directions, respectively; τ_{rz}^c is the shear stress; and w^c and u^c denote the vertical and radial deflections, respectively. The superscript c indicates responses for concentrated load; ν and E are Poisson's ratio and elastic modulus, respectively.

The responses under uniform contact pressure q distributed over a circular area of radius a are then derived from the integral:

$$S = q\frac{a}{H}\int_0^\infty \frac{S^c}{m} J_1\left(m\frac{a}{H}\right)dm \tag{2}$$

where J_1 is a Bessel function of first kind and order one; $S^c = \{\sigma_z^c, \sigma_r^c, \sigma_t^c, \tau_{rz}^c, w^c, u^c\}$ denotes the responses under a concentrated load; and $S = \{\sigma_z, \sigma_r, \sigma_t, \tau_{rz}, w, u\}$ is the response under a circular distributed load q with radius a.

Solutions of layered linear viscoelastic problems, as shown in Figure 1, can be derived from the corresponding linear elastic solutions by using the elastic–viscoelastic correspondence principle which states that for homogeneous and isotropic materials, the Laplace transform of the constitutive equations for linear viscoelastic problem can be formulated in a similar manner as for linear elastic problems (Findley et al. 1976, Huang 2004, Kim 2011).

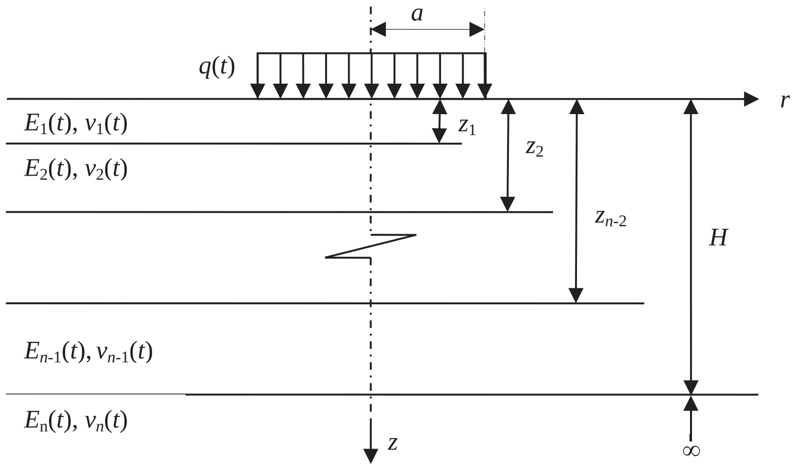

Fig. 1. Layered linear viscoelastic system subjected to circular distributed load.

The strains for linear viscoelastic (LVE) materials under external loading are time-dependent and varies therefore continuously. The linear incremental strain at time τ can now be written as:

$$d\varepsilon(\tau) = \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \tag{3}$$

and this leads to the hereditary (convolution) integral of LVE theory as:

$$\sigma(t) = \int_0^t E(t-\tau)\frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \tag{4}$$

Applying the Laplace transform of Equation (4) leads to:

$$\hat{\sigma}(s) = L(\sigma(t)) = L\left(\int_0^t E(t-\tau)\frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau\right) = s\hat{E}(s)\hat{\varepsilon}(s) \tag{5}$$

where $\hat{E}(s)$ and $\hat{\varepsilon}(s)$ denote the Laplace transform of the modulus and the strain functions, respectively. Equation (8) has the form of Hooke's law; thus, LVE problems can be considered as elastic problems in a Laplace domain given that the stiffness E and the Poisson's ratio v are replaced by $s\hat{E}(s)$ and $s\hat{v}(s)$, respectively.

The challenging task when using the elastic-viscoelastic principle for solving a layered LVE system is to obtain the Laplace inversion (Ahmed and Erlingsson, 2015). Here a numerical inverse Laplace transform based on the algorithm by Abate and Valko (2004) has been used.

Huang (2004) has in the KENLAYER code used a different approximating approach where Prony or Dirichlet series are used to model the responses at a number of selected time durations (called collocations nodes). The LVE response at time t is then given by:

$$S(t)\sum_{i=1}^{n} b_i \exp\left(\frac{-t}{T_i}\right) \tag{6}$$

where $S(t)$ is the LVE response at time t; T_i is the retardation time; n is the number of retardation times selected; and b_i are the coefficients obtained from the known values of the responses at selected time durations.

Furthermore, in the current study an approximate method based on collocation of LVE responses at selected time durations was employed so as to enhance the computational performance of the elastic-viscoelastic correspondence principle for analysis of pavement structures subjected to moving loads, as discussed in the subsequent sections. In this study, 11 time durations of 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1, 3, 10, 30, and 100 sec were used, as recommended by FHWA (1978), and retardation times T_i of 0.01, 0.03, 0.1, 1, 10, 30, and $\propto$ sec were adopted.

The three alternatives for LVE solutions have been implemented using a layered elastic program (Erlingsson and Ahmed 2013).

3. Extensions of stationary LVE solutions for Moving Load Problems

Superposition of stationary LVE solutions can be applied to obtain the responses of flexible pavement structures subjected to moving loads, as shown in Figure 2 (Papagiannakis et al. 1996, Huang 2004, Kim 2011).

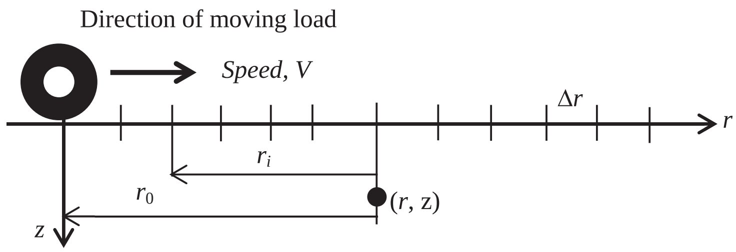

Fig. 2. Moving load.

Mathematically the superposition can be expressed as:

$$S_{moving}(z,r,t_n) = S(z,r_0,t_n - t_0) + \sum_{i=1}^{n}\left[S(z,r_i,t_n - t_i) - S(z,r_{i-1},t_n - t_i)\right] \tag{7}$$

where S_{moving} is the response of the pavement under a moving load at the current time t_n; S is the stationary LVE response; z is the depth; r_0 is the distance to the starting point of the moving load; r_i is the distance to grid location i; t_0 is the starting time; and t_i is the time when the load is at location i.

This formulation is computationally expensive. Therefore a simplified approach has been introduced, by assuming a time dependent loading in the form of a sine-squared function as (Papagiannakis et al. 1996, Huang 2004):

$$P(t) = A\sin^2\left(\frac{\pi}{2} + \frac{\pi}{D}t\right) \tag{8}$$

where $D = 12a/V$ is the duration of the load in sec; a is the contact radius; V is the speed of the moving load; A is the amplitude; and t, $(-D)/2 \le t \le D/2$, is the time in sec.

4. Verification of Viscoelastic Solutions for Stationary and Moving Load

The above approaches have been used to analyse a four layered viscoelastic pavement structure which was reported by Kim (2011). The geometry of the layered structure over a rigid bottom as well as the wheel loading are shown in Figure 3. The creep compliance of the asphalt layer was given in the form of a power function.

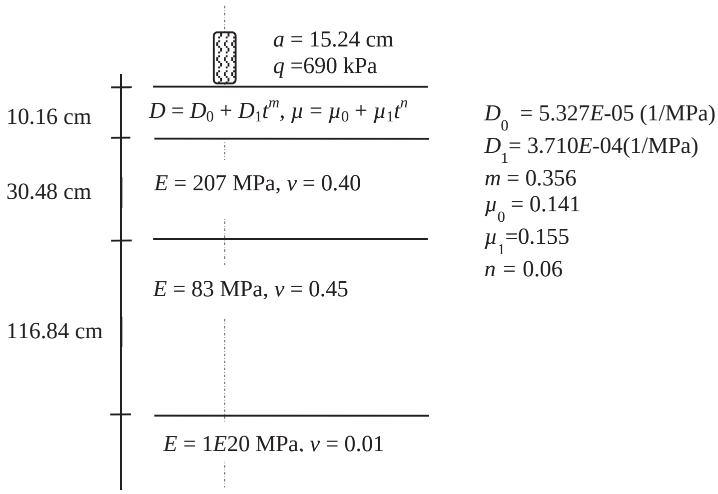

Fig. 3. Four layered viscoelastic system.

Figures 4 and 5 present the results of the LVE analysis using the three approaches, namely, based on the elastic-viscoelastic correspondence principle (denoted LVE in the figures), based on the collocation method using the viscoelastic responses at 11 time durations (denoted LVE Collocation in the figures), and based on the collocation method using elastic responses at 11 time durations (denoted LE Collocation in the figures). The LVE responses from the correspondence principle produced very similar results, as reported in Kim (2011). The results indicate that the approximate solutions agreed well with the exact solutions from the correspondence principles. Furthermore, solutions obtained from the KENLAYER code are also included in the figures for verification and as expected agreed perfectly with the LE collocation as similar implementation was used in KENLAYER.

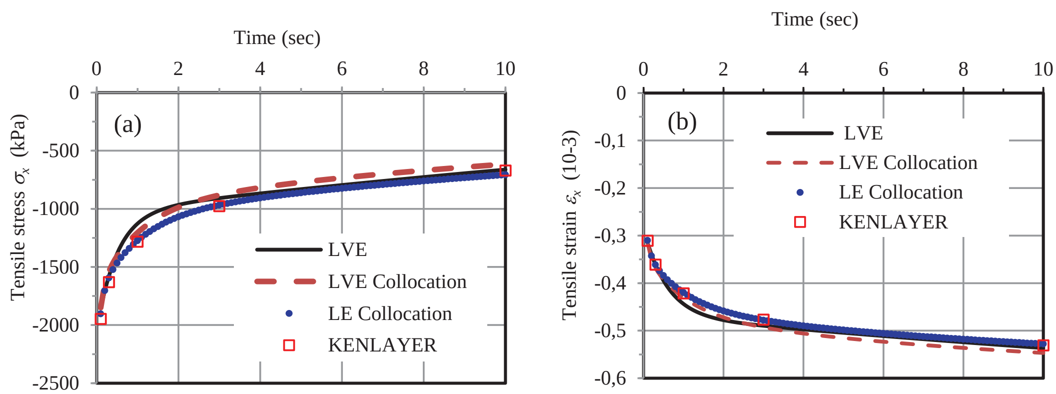

Fig. 4. (a) Tensile stress and (b) tensile strain at the bottom of the asphalt layer under the centre of the wheel.

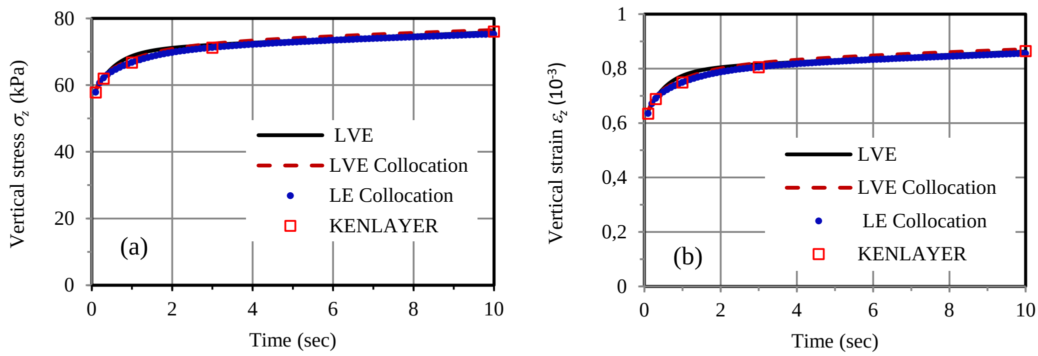

Fig. 5. (a) Vertical stress and (b) vertical strain at the top of the subgrade under the centre of the wheel.

Furthermore, the structure in Figure 3 was also analysed for a moving load of the same magnitude as in Figure 4 at a travelling speed of 60 km/h. The analysis was made using the direct correspondence principle and the LVE collocation approach. The results were further compared with results obtained from the simplified approach using the VESYS model. Figure 7 presents the longitudinal tensile stress at the bottom of the asphalt layer. The results agreed well with those reported in Kim (2011).

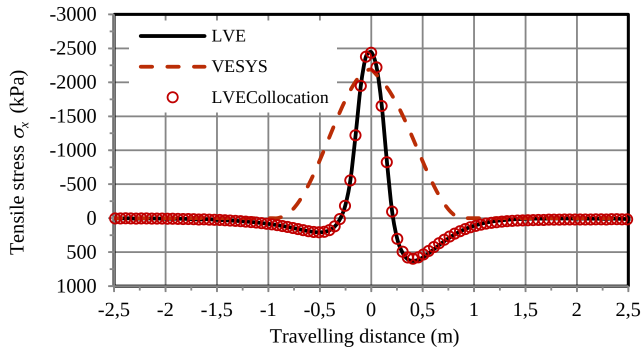

Fig. 6. Longitudinal tensile stress at the bottom of the asphalt layer under a moving load.

5. Comparison of LVE Responses with Full Scale APT Measurements

Results from a Full Scale Accelerated Pavement Test (APT) using a Heavy Vehicle Simulator (HVS) have been analysed to verify the numerical approach against real response measurements. A detailed test description can be found in Wiman and Erlingsson (2008) and Saevarsdottir et al. (2014). The test structure, named SE10, was composed of five layers; asphalt concrete wearing course (ABT16), bituminous base course (AG32), unbound crushed rock base, subbase, and fine sand subgrade over a stiff underlying structure. Figure 8 shows the cross-section of the test structure. The test structure was instrumented in order to measure the longitudinal and transversal horizontal strains at the bottom of the asphalt layer. Four H-shaped asphalt strain gauges were installed to measure the responses (Wiman 2010).

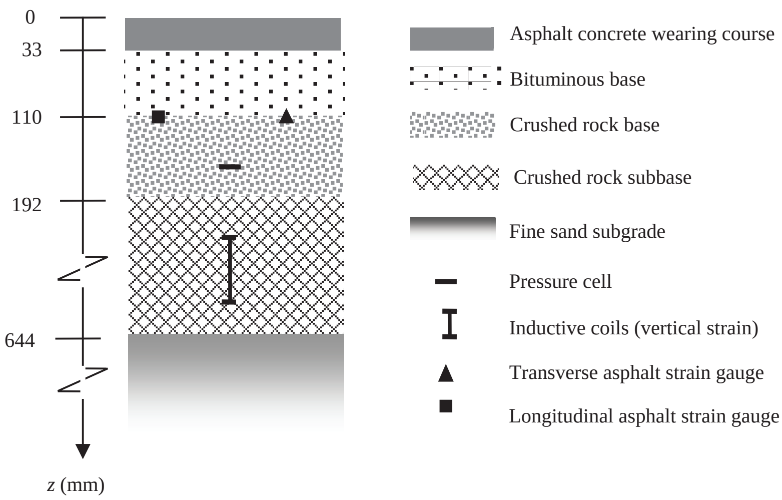

Fig. 7. Cross section of the SE10 HVS test structure.

The resilient modus of the crushed rock base, subbase and fine sand subgrade were back-calculated from falling weight deflectometer (FWD) measurements performed on the surface using plate loads of 30, 50 and 65 kN with a plate diameter of 30 cm, see Table 1.

Table 1. Elastic material properties for the base, subbase and subgrade layers of the SE10 pavement structure.

Layer	Stiffness, E (MPa)	Poisson's ratio, *v*(-)
Crushed rock base	245	0.35
Subbase	100	0.35
Fine sand subgrade	55	0.35
Massive bedrock	5171	0.35

The dynamic modulus data for the bound layers were estimated in the laboratory. Figure 9a) and b) present the master curves for a dynamic modulus (using a sigmoidal fitting function) and the phase angle of the two asphalt mixtures.

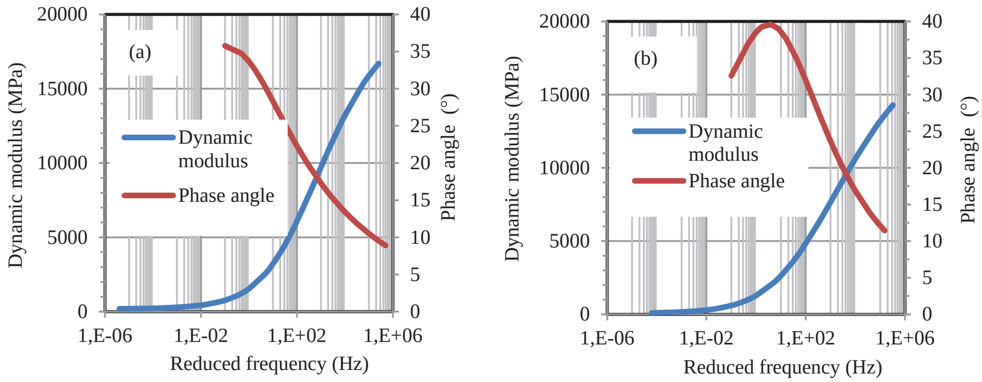

Fig. 8. Master curve for dynamic modulus and phase angle of (a) ABT16 and (b) AG32 mixes.

The asphalt mixtures creep compliance has been derived from the complex modulus data based on numerical methods documented in the literature (Findley et al. 1976, Baumgaertel and Winter 1989, Tschoegl 1989) in the form of a power function ($D(t) = D_0 + D_1 t^m$) or:

$$
\begin{aligned}
&D'(\omega) = D_0 + D_1 \Gamma(m+1)\omega^{-m} \cos(m\pi(2k+1/2)) \\
&D''(\omega) = D_1 \Gamma(m+1)\omega^{-m} \sin(m\pi(2k+1/2)) \\
&k = 0, \pm 1, \pm 2 \ldots
\end{aligned}
\tag{9}
$$

where $D'(\omega)$ and $D''(\omega)$ are the storage and loss creep compliance, respectively, at frequency ω. D_0, D_1 and m are the power function parameters for creep compliance and Γ is the gamma function. In this study an algorithm outlined by Kim et al. (2008) was used to obtain the parameters D_0, D_1 and m for the asphalt mixtures. Thereafter the creep compliance at 11 time durations was obtained from the power function in order to estimate the Dirichlet series parameters for the creep compliance. Table 2 presents the power function parameters for creep compliance of the different asphalt mixes.

Table 2. Power function parameters for creep compliance of the asphalt mixes.

Mix	Temperature (°C)	Parameters		
		D_0 (1/MPa)	D_1 (1/MPa)	m (-)
ABT16	0	5.03E-05	1.47E-04	0.313
	10	5.05E-05	4.45E-04	0.346
	20	1.47E-07	1.27E-03	0.312
AG32	0	5.04E-05	2.11E-04	0.274
	10	5.10E-05	5.83E-04	0.312
	20	5.06E-05	1.55E-03	0.337

The results of the calculation along with measurements are given in Figures 9–10. Figure 9a) and b) presents the measured and calculated longitudinal and transverse tensile strains at the bottom of the bituminous base layer under the centre of one wheel for T = 10 °C for a dual wheel assembly. The measured and LVE calculated longitudinal tensile strain were slightly asymmetric about the peak value, indicating some viscoelastic behaviour. For W = 60 kN, the calculated LVE responses agreed very well with the measured values. However, for the 80 kN wheel load, the LVE calculated peak longitudinal strains surpassed the measured value by about 10%.

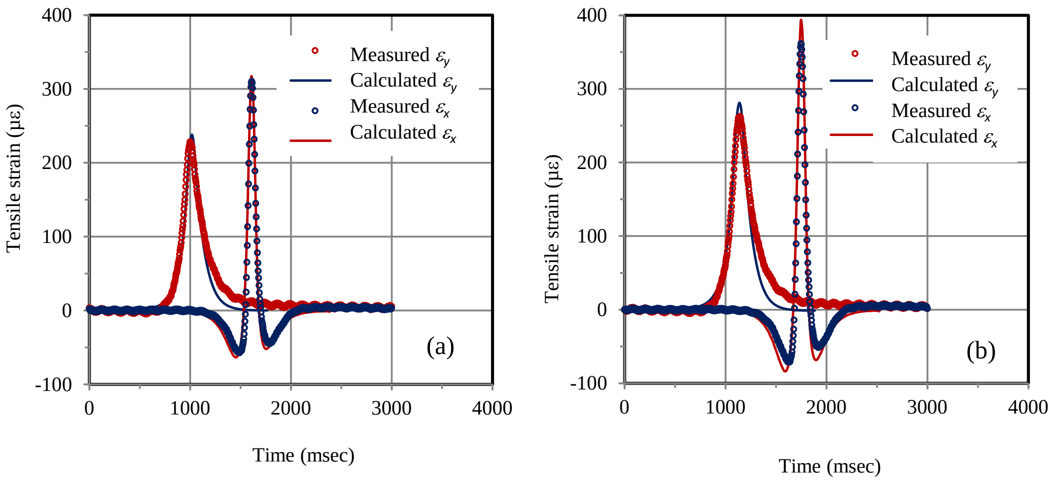

Fig. 9. Longitudinal and transversal strains at the bottom of the bituminous base under the centre of one wheel for (a) W = 60 kN and (b) W = 80 kN at T = 10 °C.

Figure 10a) to d) presents the measured and calculated longitudinal and transversal tensile strains for wheel loads of 30, 50, 60 and 80 kN at the bottom of the bituminous base layer under the centre of one wheel for T = 20 °C for a dual wheel assembly. The measured and LVE calculated longitudinal tensile strain were not symmetric about the peak, indicating that the structure exhibited considerable viscoelastic behaviour. Similar to the responses at 10 °C, very good agreement between the calculated and measured LVE responses were observed except for the 80 kN wheel load where the calculated peak longitudinal strain response slightly exceeded the measured values.

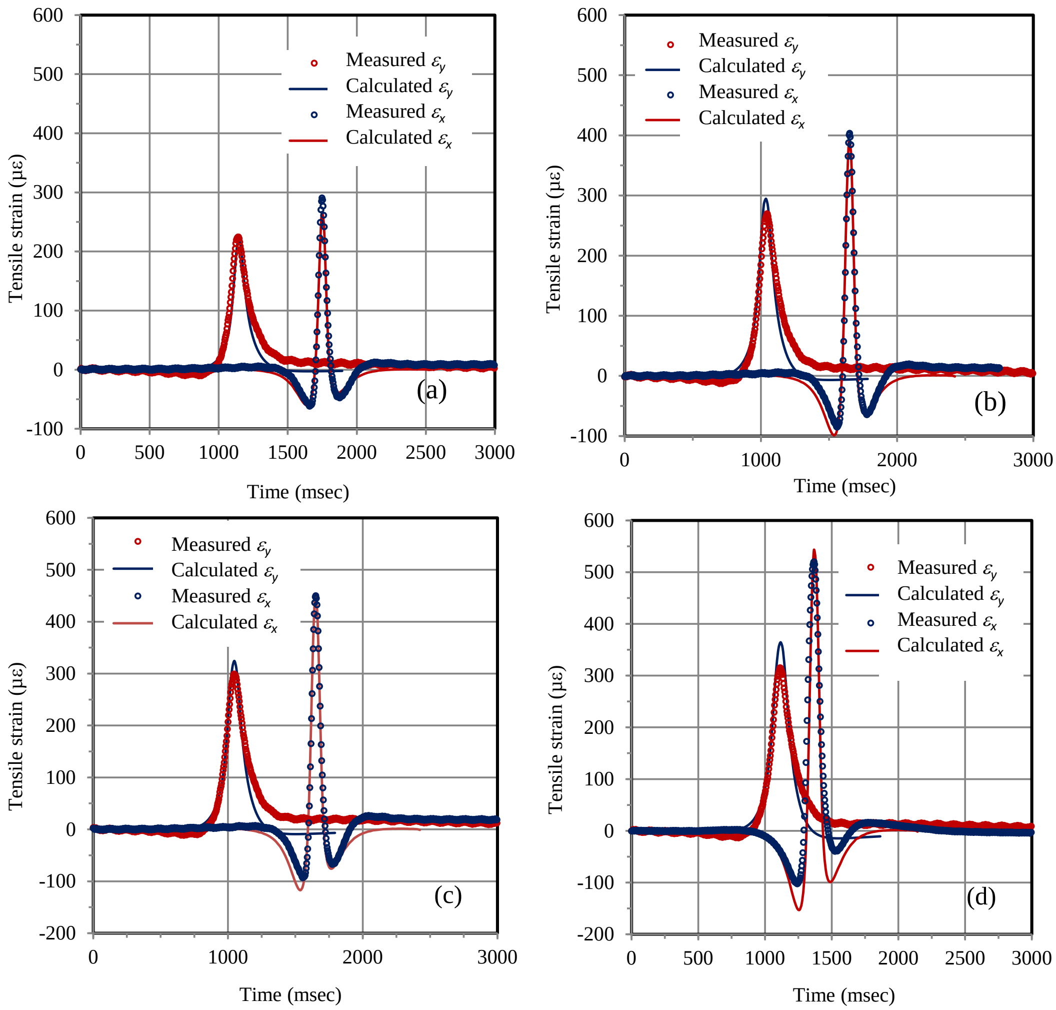

Fig. 10. Longitudinal and transversal strains at the bottom of the bituminous base under the centre of one wheel for (a) W = 30, (b) W = 50, (c) W = 60 and (d) W = 80 kN at T = 20 °C.

6. Conclusions

This paper demonstrated the application of layered viscoelastic analysis to estimate flexible pavement structural responses based on the elastic-viscoelastic correspondence principle, using a numerical inverse Laplace transform. The paper also compared the direct layered viscoelastic solutions with a simplified approximate solution using a collocation of elastic solutions at predefined time durations and it was observed that the elastic collocation method gave a good approximation. Furthermore, the paper illustrated the use of the collocation method using LVE solutions at selected time durations (LVE collocation) in order to improve the accuracy, as compared to elastic collocation (LE collocation). Extension of the LVE collocation method for solutions of moving loads was also demonstrated and the results were compared with the VESYS and the direct elastic-viscoelastic correspondence principle and indicated very good agreement.

Finally, the LVE collocation approach for moving loads was employed to analyse a test pavement structure subjected to moving dual wheel loads of magnitudes 30, 50, 60 and 80 kN using an HVS. From the comparison of the measured and computed longitudinal and transversal strain LVE responses, it was observed that the LVE calculated responses agreed well with the measurements.

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